LISBOA SCHOOL OF ECONOMICS & MANAGEMENT

MASTER IN FINANCE

MASTER'S FINAL WORK

PROJECT WORK

FITTING THE TERM STRUCTURE OF YIELD SPREADS

VELMA DE JESUS RODRIGUES

SEPTEMBER - 2014

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Abstract

This study aims to fit and analyze the behavior of the Yield Spread curve in the context of Portugal Government Bonds, covering a period of January 2004 through June 2014, when Portugal faced a liquidity and debt crisis. In order to extract the Yield Spread curve, we use a disjoint method. This method requires as an input both a defaultable and non-defaultable term structure: we use the default-free curve estimated by the ECB and the defaultable term structure is estimated by the Nelson-Siegel model (1987). Due to the important role that forecasting plays in understanding how term structure evolves, the secondary objective of this work is to forecast the yield curve by predicting the parameters of Nelson-Siegel model (1987) using the Random Walk with drift as the benchmark model and the AR(1) and the VAR(1) model as competitors models. The results include the empirical analysis of Portuguese Government yield spread curve and, concerning the yield curve forecasting, we conclude that AR(1) and VAR(1) slightly outperformed the benchmark model and these models performance improves as the forecasting time horizon increases.

Keywords: Yield Curve, Yield Spread, Nelson-Siegel Model, Forecasting, Term Structure

List of Abbreviations

ACT/ACT - Actual/Actual convention

AIC -Akaike's Information Criterion

AR(1) - Autoregressive Process of Order 1

BIC - Bayesian Information Criterion

ECB - European Central Bank

EMU - Economic Monetary Union

ICMA - International Capital Market Association

ISIN - International Securities Identification Number

OLS - Ordinary Least Squares

GLS - Generalized Least Squares

RW - Random walk

VAR(1) - Vector Autoregressive Process of Order 1

List of Tables

List of Figures

Contents

1 Introduction

In the recent past, the financial industry as a whole has changed rapidly. The rapid development of government bond markets in Euro-zone may be explained by the effects of globalization and the process of European Monetary integration. Although the creation of European Monetary Union theoretically eliminates the exchange rate risk, yield spread still exists due to liquidity issues and country-specific default risk explained by the adoption of a certain fiscal policy and external shocks. So, understanding the evolution of yield spread is a crucial subject in finance, since the historical spreads may give to investors an overview about market expectations, potential investment opportunities or how to hedge portfolios. A proper yield spread evolution assessment will bring value to fixed income managers and, since the yield spread is a source of risk, it must be considered in financial risk management, pricing financial products, fiscal debt, portfolio allocation, or even in monetary policy implementation.

Given the importance of this topic, there are several empirical literature that focus on the time-series dynamics of yield spread. Duffee (1999) and Elton et al. (2001) both fitted the credit spread using non-callable corporate bonds. The empirical research of Elton et al. (2001) investigates the existence of risk premium in corporate bonds spread and they conclude that the credit spread may be explained by the expected default loss, tax premium and risk premium. Since a large part of existence empirical research on fitting the yield spread of corporate bond relies on non-callable bonds, Berndt (2004) extracted the credit spread from prices of callable corporate bonds by treating simultaneously the effect of credit risk and optionality. There are also empirical studies based on the extraction of Government yield spread, such as, Dullman and Windfuhr (2000), Geyer et al. (2003) and Duffie et al. (2003). The research of Dullman and Windfuhr (2000) investigates the dynamics of the yield spread between Italian and German Government bonds after the exchange-rate agreement in 1998. Geyer et al. (2003) deducted the yield spread of Government bonds issued by member states of EMU and they noted a significant and volatile credit spread between German Bund yields and the others member states of EMU yields. Duffie et al. (2003) developed a model, under the framework of Duffie and Singleton (1999), that estimate the term structure of credit spread using the Russian dollar-denominated bonds, which takes into consideration the risks of default, risk of restructuring and compensation for lack of liquidity.

Regarding the term structure modeling, there are also a vast literature concerning this field. The most used approach among the financial market practitioners and central banks is the class of Nelson-Siegel Models (1987), which is used to fit the term structure of interest rates through a flexible and smooth parametric function. In order to increase flexibility in the standard Nelson-Siegel model (1987), Svensson (1994) proposed to add a second hump-shape factor with its own decay parameter. However, a multicolinearity problem may arise by adding a second decay parameter. To overcome the multicolinearity issue Pooter (2007) proposed an Adjusted Svensson model. Note that the original Nelson-Siegel model is a static model, which fits the cross section of interest rates at a given point in time. That is why Diebold and Li (2006) suggested a dynamic version of Nelson-Siegel model to fit the yield curve.

Despite the progress in the term structure of interest rate modeling, the practical estimation of yield spread field using Portugal empirical data has been underexplored. This study tries to fill this gap by fitting the yield spread curve in the context of Portuguese Government Bonds. The latter yield spread curve is computed as the difference of Portuguese Yield Curve, which is estimated using the standard Nelson-Siegel model (1987), and the yield curve of AAA-rated countries in euro area estimated by ECB, which is considered as the benchmark zero-coupon yield curve.

Since this research covers the period that Portugal was under the Economic and Financial Adjustment Programme, besides the yield spread curve behavior analysis, is also addressed the impact of the financial bailout.

Much of this research focuses on fitting the Portuguese Treasury Bonds yield curve and consequently the extraction of its yield spread. However, many tasks such as pricing financial instruments, portfolio diversification or risk management, require not only the term structure of interest rates but also the knowledge of how the term structure will evolve plays an important role in finance. So, the secondary aim of this research is to better understand the dynamic evolution of the yield curve. To meet the latter objective, is performed the forecasting of the yield curve using three different models: Random Walk with drift, AR(1) and VAR(1).

The remaining of the text is organized as follows. Section two provides a brief discussion of the theoretical framework, where some basic concepts are highlighted. Section 3 presents the methodology used to fit the term structure of yield spread as well as the data chosen to achieve this purpose. Section 4 analyses the empirical in-sample and out-of-sample results. Lastly, the Section 5 stresses the conclusions of this study.

2 Theoretical Framework

Here is introduced some basic concepts prior to discussing in detail the term structure model used to generate the yield curve in this research. Afterwards, is explained the approach used to extract the yield spread curve as well as different models used in the yield curve forecasting.

2.1 Term Structure: Basic Concepts

The term structure of interest rate depicts a series of interest rates as a function of time to maturity. Further, there are three equivalent theoretical representation of term structure: yield curve, forward curve and discount curve. As the latter curves names suggest, they graphically depict a series of yields, forward rates and discount factors as a function of maturity, respectively.

Regarding the yield curve, $y_t(\tau)$, it can be computed as an equally-weighted average of instantaneous forward rates. This sort of rate is defined as forward rate, $f_t(\tau)$, with the limit of maturity tending to zero. More precisely, it can be interpreted as a forward rate marginal cost for a very short period of time. So, the yield curve or spot rate curve, with τ periods to maturity, is given by:

$$y_t(\tau) = \frac{1}{\tau} \int_0^{\tau} f_t(m)dm \tag{1}$$

This important relationship between yield and forward rate is a critical point of the Nelson-Siegel class of model.

The discount curve, $P_t(\tau)$, as mentioned before, is a function that depicts the relationship between the discount factors and term to maturity and it can be obtained from the yield curve by the following relation:

$$P_t(\tau) = \exp[-\tau y_t(\tau)] \tag{2}$$

Therefore given the relations mentioned above, we can find the following link between these three curves:

$$f_t(\tau) = -\frac{1}{P_t(\tau)}\frac{dP_t(\tau)}{d\tau} = y_t(\tau) + \tau\frac{dy_t(\tau)}{d\tau} \tag{3}$$

Once the yield curve or the forward curve is known, any coupon bearing bond can be priced as a sum of discounted cash flows. Though, the three term structures representation mentioned before are not directly observed in the market for an extensive range of maturities. Saying so, these curves must be estimated from bond prices observed in the market using theoretical models to fit the term structure, such as the Bootstrapping of Fama and Bliss (1987), cubic spline function of McCulloch (1975), exponential splines of Vasicek and Fong (1982), parametric function suggested by Nelson-Siegel (1987), and later extended by Svensson (1994), Bjork and Christensen (1999), Pooter (2007) or non-parametric model introduced by Linton et al. (2001). There are several studies that empirically compare these different estimation methods, such as Bliss (1997) and Jeffrey et al. (2006).

2.2 Nelson-Siegel Model (1987)

Nelson and Siegel (1987) proposed a three parameter exponential-polynomial function. This approach fits the forward curve with:

$$f_t(\tau) = \beta_{0,t} + \beta_{1,t}\mathrm{exp}\left(-\frac{\tau}{\lambda_t}\right) + \beta_{2,t}\left(\frac{\tau}{\lambda_t}\right)\exp\left(-\frac{\tau}{\lambda_t}\right) \tag{4}$$

This expression can be viewed as a constant plus a Laguerre function, which is a product between a polynomial and an exponential decay term.

As described in the previous subsection, the yield curve is obtained by an equally-weighted average of instantaneous forward rates. So the spot yield curve is given by:

$$y_t(\tau) = \beta_{0,t} + \beta_{1,t}\left[\frac{1-\exp\left(-\frac{\tau}{\lambda_t}\right)}{\frac{\tau}{\lambda_t}}\right] + \beta_{2,t}\left[\frac{1-\exp\left(-\frac{\tau}{\lambda_t}\right)}{\frac{\tau}{\lambda_t}} - \exp\left(-\frac{\tau}{\lambda_t}\right)\right] \tag{5}$$

Thus, the shape of the yield curve is given by the sum of three components, each of them resulting from the product of weighting functions and the model parameters $\beta_{0,t}$, $\beta_{1,t}$ and $\beta_{2,t}$. Each component can be interpreted as follows. The component β_0 is multiplied by one for all maturities, so it can be interpreted as a long-term component. The component of β_1 is the short-term component, since its weighting function starts at 1 and afterwards will decay rapidly to zero. Lastly, in the component of β_2, the weighting function starts at zero, increases for medium maturities and then decays to zero again, which produce a hump-shaped curve. Moreover, yields converge to β_0 if maturity tends to infinity and converge to the sum of β_0 and β_1 if maturity tends to be an infinitely short period of time. Thus, $\beta_0 + \beta_1$ can be defined as instantaneous interest rate. To ensure that yields are positive, β_0 and

$\beta_0 + \beta_1$ must be positive. Note that the λ_t parameter determines the rate of decay and also at which maturity the medium-term component reaches its maximum.

The basic parametric model of Nelson-Siegel (1987) is a static approach, which gives an estimation of yield curve at a certain point in time. However, Diebold and Li (2006) suggest a dynamic version of Nelson-Siegel, where the dynamics factors β_0, β_1 and β_2 can be viewed as level, slope and curvature of yield curve, respectively. Besides that, in the new dynamic framework they propose to fix the decay parameter λ at a pre-specified value in order to ease the estimation process. More precisely, if λ is fixed, the nonlinear measurement equations become linear and the estimation proceed using the cross-sectional OLS procedure.

It is also worth noting that Nelson-Siegel model has enough flexibility to capture a range of shapes of yield curve observed in the market (such as monotonic, humped and S-shaped curve) and its dynamic version is able to replicate the stylized facts of yield curves such as: yield curve are normally upward sloping, when yields reach high (low) levels tend to decrease (increase) in the next moment - *behavior of mean reversion*, shorter term yields are more volatile than the longer term yields, long-term yields are more persistent than the shorter term.

2.3 Modeling the Yield Spread

With respect to the literature about the yield spread modeling, there are two different parametric models: disjoint and joint method.

Regarding the disjoint method, the basic idea is deriving both the term structure of nondefault zero-coupon yields and the term structure of risky zero-coupon yields separately. Thus, the term structure of zero-coupon yield spread is obtained by the difference between the risky zero-coupon yield curve and the benchmark zero-coupon yield curve. In short, it is a three-step procedure. Dullman et Windfuhr (2000) empirically computed the yield spread between German and Italian Sovereign Bonds as the difference between the yields of Italian and German bonds. Annaert (2000) and Landschoot (2004) both modeled the European credit spread as the difference between the yield of the corporate bond and the yield of government bond. However, Annaert (2000) made this subtraction with respect to the bonds with same average duration and Landschoot (2004) divided the term structure of yield spread by rating categories. Elton et al. (2001) empirically computed the yield spread curve as the difference between the yield of the zero-coupon corporate bond and the yield of zero-coupon government bond of the same maturity.

The joint estimation framework, proposed by Houweling et al. (2001), consists in jointly estimating both term structure of zero-coupon yields simultaneously. In other words, since a defaultable term-structure comprises in a default-free curve (proxied by a theoretically riskless government curve) and in a yield spread curve, the basic idea of this approach is a decomposition of the defaultable term structure into these two features. By saying so, this method focuses in modeling the spread and the default-free is considered as a part from the government curve. In short, all the parameters in the model for the discount spread function and the government curve are simultaneously estimated from a combination of both data set and performing a non-linear GLS procedure.

2.4 Forecasting the Yield Curve

Modeling the yield curve is an important field in financial economics, so the forecasting. Many tasks such as pricing financial instruments, portfolios allocation, managing financial risks require not only the current term structure of interest rates but also the understanding how the yield curve may evolve. The results of the forecast also may serve as a guideline for strategic plans concerning the fiscal debt structuring and monetary policy.

In the context of the Nelson-Siegel model, time series models can fit to the history of β parameters and used to forecasts. To accomplish the latter aim, the out-of-sample forecasting will be performed using three different models: Random Walk with drift, AR(1) and VAR(1).

The Random Walk is the simplest model. It is defined as a process where the future value is given by the previous value plus an error. The error term is defined as a white noise process, which is formed by a sequence of uncorrelated variables with zero mean and constant variance. This process has the following form:

$$y_t(\tau_i) = y_{t-1}(\tau_i) + \varepsilon_t(\tau_i), \quad \varepsilon_t(\tau_i) \sim N(0, \sigma^2(\tau_i)) \tag{6}$$

and it implies that all the expected future values of $Y_{t+\delta}$, $\delta > 0$, is given by Y_t.

Whenever the time series present an average upward or downward trend, it is expected that the forecasted values continue to exhibit a trend. In this case, a non-zero constant trend is added to the random walk and the process is represent as follows:

$$y_t(\tau_i) = \alpha + y_{t-1}(\tau_i) + \varepsilon_t(\tau_i), \quad \varepsilon_t(\tau_i) \sim N(0, \sigma^2(\tau_i)) \tag{7}$$

Note that for $\alpha > 0$ the time series will exhibit an upward trend and, consequently, $\alpha < 0$ will denote a downward trend.

The random walk with drift process is particularly relevant since many financial time series seem to follow a pattern that exhibits a trend.

The AR(1) is an univariate autoregressive model of first order and is represented as follows:

$$y_t(\tau_i) = \theta(\tau_i) + \phi(\tau_i)y_{t-1}(\tau_i) + \varepsilon_t(\tau_i), \quad \varepsilon_t(\tau_i) \sim N(0, \sigma^2(\tau_i)) \tag{8}$$

Actually, it is a linear regression model, which the forecasted value depends on its own past values. In this case, since there is only one lag in the autoregressive process, the expected future value is given by the previous value plus the $\theta(\tau_i)$. The value of ϕ defines whether the time series is stationary or not. If $|\phi| > 1$, the process is non-stationary and it grows exponentially. If $|\phi| < 1$, the process is stationary. Once again, the error term is formed by a series of random shocks and it is uncorrelated with the past values of the AR series. Note that when $\phi(\tau_i)$ takes the value of 1 this becomes a random walk process.

The VAR(1) is a vector autoregressive process of first order, which may be used to forecast multivariate time series. This process is given by:

$$y_t = \mu + \Gamma F_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim N(0, \Sigma) \tag{9}$$

Where F_{t-1} represents the factor matrix.

The VAR(1) assumes that the ε_t are uncorrelated through time and independent and identically distributed.

The main difference between the last two forecasting models is, in AR(1) each parameter of the Nelson-Siegel (1987) model is regressed against its own past values. In the other hand, in VAR(1) is created a matrix where each parameter forecasted is regressed against its own past values and against all other parameters included in the system.

Regarding the empirical research of this field, there are several authors that investigated the quality of forecasting models based on Nelson Siegel model (1987). Diebold and Li (2006) used several forecasting model such as Random Walk, Slope Regression, Fama-Bliss forward rate regression, Cochrane-Piazzesi (2002) forward curve regression, AR(1) and VAR(1) on yield levels and VAR(1) on yield changes. By assessing the quality of the forecast based on dynamic version of Nelson-Siegel, Diebold and Li (2006) argued that their forecasting approach produced accurate results, outperforming several benchmark forecasting model, especially for a 6 and 12-months forecast horizon. Pooter (2007) used the benchmark Random Walk model, AR(1) and VAR(1) and the empirical results suggest that the four-factor model, which is the model that adds a second slope to the Nelson-Siegel model, produces forecast that outperform the benchmark Random Walk. Molenaars et al (2003) results suggest that the forecasting models used, which are the AR(1) and dynamic Nelson-Siegel model, underperform the random walk model. Further, there is no advantage of applying a model that is more complex that AR(1) to forecast.

3 Data and Methodology

In this section is provided a detailed discussion regarding the data used in this research as well as the procedure followed to fit the term structure of yield spread. Hereafter, will be explained how the forecasting is performed.

3.1 Data Description

This research uses daily prices of Government bonds of varying maturities issued by Portugal over the period January 1^{st} 2004 through June 30^{th} 2014, providing ten years and six months of daily data, taken from *Bloomberg* database. Besides daily bonds prices, the data set includes for each bond considered the issue date, maturity date, ISIN, first coupon payment date, coupon rate, date count convention and the amount issued.

In order to avoid distortions in the estimation, several filters are considered to determine the sample of bonds. Similar to Diebold and Li (2006), we exclude bonds with liquidity issues, which are Treasury Bills with less than one month to maturity and bonds with less than one year to maturity, bonds with option features and perpetual bonds are excluded as well.These filters, which are also applied by the Center for Research in Security Prices, leaves us with a data set of 116 bonds.

For yield spread fitting purpose, besides the Portuguese yield curve, which is considered as the term-structure of risky zero-coupon yield, we also need a nondefault zero-coupon yield curve. So, as a benchmark zero-coupon yield curve, is considered the government bonds yield curve of AAA-rated countries in euro area estimated by ECB using the Svensson model (1994). Note that the ECB considered the rating provided by Fitch Ratings. Using this AAA-rated yield curve, we extract the yield for maturities 3-months, 6-months, 9-months, 1-year, 15-months, 18-months, 21-months, 2-years, 30-months, 3-years, 4-years, 5-years, 6-years, 7-years, 8-years, 9-years, 10-years, 15-years and 20-years. We select these maturities with the aim to analyze the behavior of yield spread throughout the maturity spectrum, which ranges from short to long-term maturities.

3.2 Methodology

3.2.1 Fitting the Yield Spread Curve

The traditional disjoint method mentioned in subsection 2.3 is used to fit the term structure of yield spread. According to this method, the defautable term structure and the riskless term structure must be estimated separately. In this study, the defaultable term structure is the Portuguese Government Bonds yield curve and as the risk-free term structure is assumed the yield curve of AAA-rated countries in euro area estimated by ECB.

The Nelson-Siegel model (1987) and Svensson model (1994) is two of the most widely used approach to model yield curves. This research uses the Nelson-Siegel model to fit the Portuguese yield curve because the Svensson model produces higher yield errors than the Nelson-Siegel approach (see Geyer and Mader 1999)) and the fit using Svensson method is a time-consuming process, Bolder and Streliski (1999) conclude that the Svensson model takes approximately four times more estimation time than Nelson-Siegel method. Besides that, Nelson-Siegel model produces a parsimonious function of yield curve and avoids over-parametrization. Since the latter model provides a continuous function, there is not necessary to apply other models to interpolate intermediate points to obtain the discounted cash flows for all maturities.

Given the filtered sample of bonds, the first step is to determine the cash-flow and the corresponding cash-flow payment date associated to each bond. With these information is possible to create a matrix that include all cash flows payments dates and the respective cash-flow of the sample of bonds. Note that each bond is identified by their ISIN. Suppose that bond j pays the cash-flow c_{ij} in the date m_{ij}; for the sample of k bonds, the matrix is given by $M = \{m_{ij}\}$ and $C = \{c_{ij}\}$ with t rows and k columns, with $i = 1,...,t$ and $j = 1,...,k$. The number of rows defined by t determines the number of cash-flows of the bond with longest maturity. The dates when the bond j does not pay any cash-flow is completed with zero.

Prior to proceed with the estimation, is also required the computation of the accrued interest, which is calculated following the day convention ACT/ACT (ICMA). Therefore, the price of the bond, p, is given by the sum of accrued interest plus the quoted market price and the theoretical price of the bond j, $\hat{p}_j$, is given by:

$$\hat{p} = \sum_{i=1}^{n} c_{ij} e^{-y(\tau_i)\tau_i} \tag{10}$$

which c_{ij} denotes the cash-flow i paid by the bond j with τ_i periods to maturity.

Regarding the yield curve estimation, by analyzing the functional form of Nelson-Siegel model (1987) described in section 2.2 we conclude that there are four parameters that must be estimated, $\theta = \{\beta_{0,t}, \beta_{1,t}, \beta_{2,t}, \lambda_t\}$ for each day t of the period analyzed. So, an optimization process is needed to obtain the accurate parameters that best fit the existing bond prices. The parameters vector of the yield function, θ, is obtained by a nonlinear minimization of residual sum of squares between the observed prices, p, and the theoretical prices,$\hat{p}$:

$$\mathrm{RSS}(\theta) = \sum_{i=1}^{n} (p_i - \hat{p})^2 \tag{11}$$

Once the parameters of Nelson-Siegel model is estimated, we obtain the Portuguese yield curve fitted from observed coupon prices.

The *termstrc* package in R (see Ferstl and Hayden (2010)), allows the search for the optimal set of starting value by applying the start parameter grid search and, then, it applies numerical optimization approaches to perform the nonlinear minimization of RSS.

Therefore, in order to fit the yield spread curve, we assume as default-free yield curve the term structure estimated by ECB that contains only the AAA-rated euro area central government bonds. By the disjoint method, the yield spread is computed as the difference between the Portuguese Governemnt Yield and the AAA-rated euro area central government bonds yield both with the same maturity.

3.2.2 Forecasting

This research predicts the Portuguese Treasury Bonds yield curve by forecasting each parameters of the Nelson-Siegel model (1987)

To accomplish this aim we, follow the framework used in Diebold and Li (2006), Pooter (2007) and Molenaars et al. (2003). These empirical researches assess the quality of different models based on Nelson-Siegel (1987) model and use the Random

Walk as the benchmark model. Duffee (2002) suggested that forecast results are more accurate under the assumption that yields follow a Random Walk model.

Therefore, we assumed as the benchmark model the Random Walk with drift due to the pattern that is observed in many economic time series, which suggests a trend component. To analyze the forecasting capacity of Nelson-Siegel model, we forecasted its parameters using AR(1) and VAR(1). For each one of the models applied, the forecasting time horizon was 1-month, 3-months and 6-months ahead.

The conclusion concerning the forecasting capacity of Nelson-Siegel model is obtained by comparing the out-of-sample results of AR(1) and VAR(1) with the benchmark model. Lastly, in order to determine which model provides a better fit, the forecasting model selection criteria computed are AIC and BIC. The AIC is computed as:

$$AIC = -2 * log(L) + 2 * k \tag{12}$$

and BIC is calculated as:

$$BIC = -2 * log(L) + k * log(n) \tag{13}$$

Which L stands for the likelihood value, k is the number of estimated parameters and n the sample size used in the estimation.

4 Empirical Results

Prior to detail and analyze through descriptive statistics and graphical representations the results of in-sample as well as the out-of-sample fit, we should stress the importance of the *data cleaning* process.

When an estimation process is made in an automatic way for a large sample as the one used in this study, it becomes difficult to control and ensure the quality of the entire output set. Therefore, it is important to adopt the *cleaning data* process, which consists in removing particular invalid data points from the output set and then confirm if the remaining data is reliable.

Diebold and Li (2006) obtained the interval values of $4.427 < \hat{\beta}_{0,t} < 12.088$, $-5.616 < \hat{\beta}_{1,t} < 0.919$ and $-5.249 < \hat{\beta}_{0,t} < 4.234$ for the parameters estimated in the context of U.S. Treasuries. Even taking into consideration the differences between the US Treasury and Portugal Government Bonds regarding the liquidity issues, amount of Treasury Bonds issued, nature of bonds and others factors that affect the estimation of yield curve; we considered that the parameters estimated with value higher than 100 is not reliable. So, is considered as outliers the dates which the absolute value of the corresponding parameters estimated are higher than 100. Particularly in this research is removed a total of 14 data points.

4.1 In-sample fit Results

In this subsection will be discussed the results of fitting the yield curve and consequently the yield spread curve.

The Figure 1 and 2 depict several yield curve fitted by Nelson-Siegel model at some specific dates throughout the period studied . As these yield curves show, the Nelson-Siegel model is capable to capture different range of yield curve shapes: flat or almost flat (e.g. yield curve on 01/04/2004 and 25/07/2006), upward sloping (e.g. yield curve on 23/09/2009, 10/09/2012 and 19/06/2014)and humped (e.g.

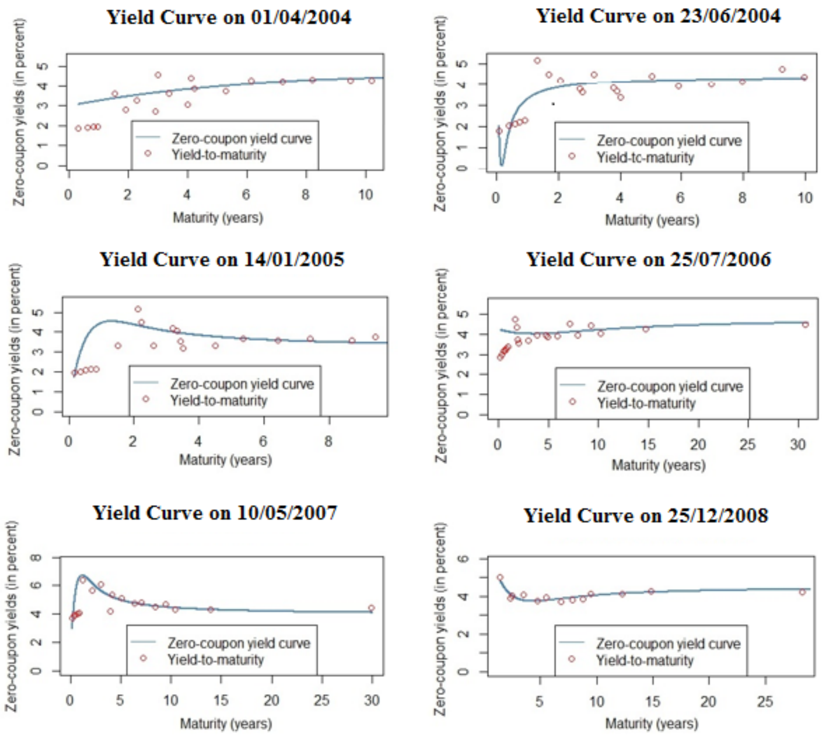


Figure 1: Selected Fitted Yield Curve

yield curve on 23/06/2004, 14/01/2005 and 10/05/2007). In this case, the yield curve depicts how the yield of the Portuguese Government Bond depends on its maturity. Note that the slope of the yield curve depicts the gap between the short and long term yields.

The Table A1 in the appendix, presents for varying maturities the descriptive statistics of the estimated Portuguese Government Bonds yield curve that covers the period from January 2004 through June 2014, such as mean yield, standard deviation, kurtosis, skewness, minimum and maximum value achieved. Through the analysis of these descriptive statistics, we can see that the yield curve on average is upward sloping since the yields increase as the maturity increases, achieving its maximum at around the maturity of 6 years. Regarding the standard deviation

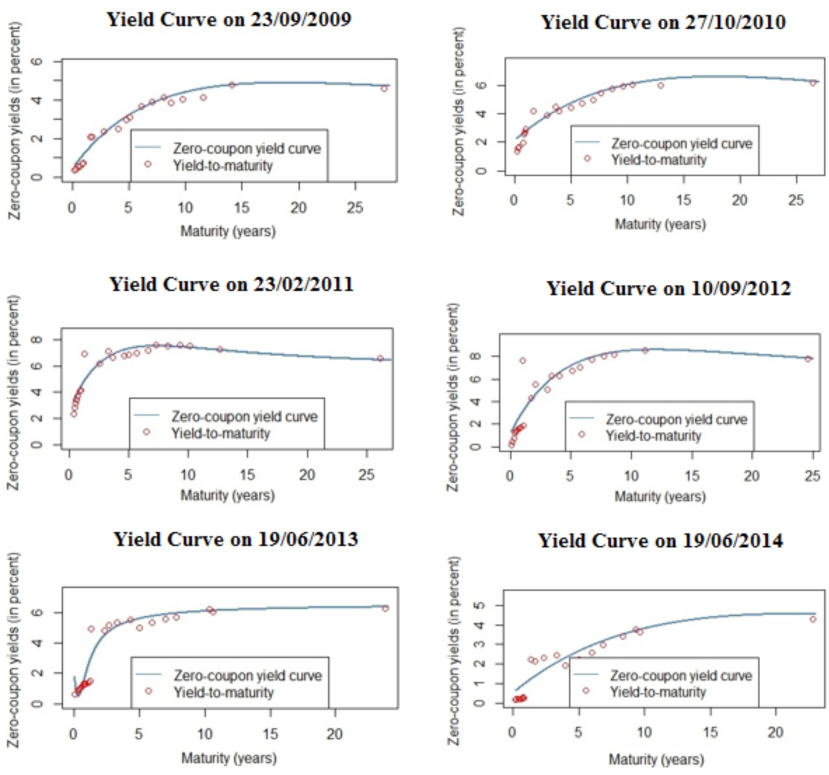


Figure 2: Selected Fitted Yield Curve (Continuation)

analysis, we conclude that on average the medium-term maturity is more volatile than the short and long-term maturity, achieving its maximum at the maturity of 5 years; this means that, at some point, the volatility decreases as the maturity increases.

The Table A2 in the appendix, details the basic descriptive statistics of the parameters β_0, β_1, β_2, λ_t estimated by nonlinear least squares for each day t of the period analyzed and in Figure 3 plots the evolution of each one of the parameters estimated.

The β_0, which can interpreted as long-term factor and defines the level of the yield curve, is, on average, positive and the yield will tends to this value as the

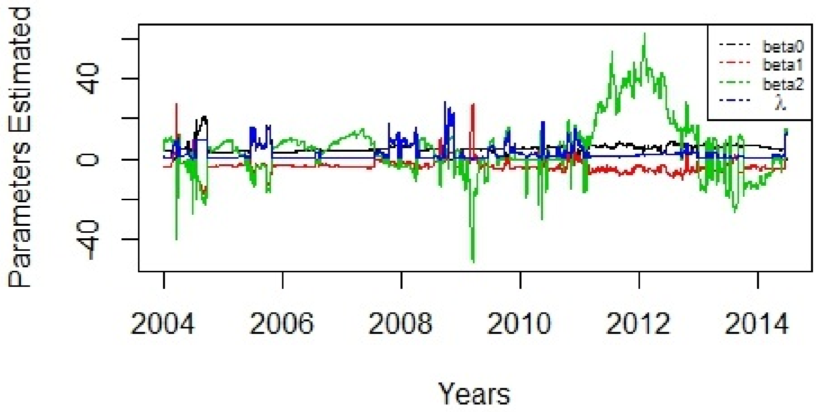


Figure 3: Evolution of Estimated Parameters over the period January 2004 through June 2014.

maturity approaches to infinite. On average, β_1 is negative, which means that throughout the period from January 2004 to June 2014 the yield curve is on average upward sloping. The β_2, on average, is also positive and specified the position of the curvature. The parameter more volatile is β_2, which can explain that the curvature of the yield curve is changing over time and in some cases the hump does not even exist (e.g. yield curve on 19/06/2014, Figure 2). It is also important to stress the ADF test done for each one of the parameters estimated. An Augmented version of the Dickey-Fuller tests the existence of unit root for a univariate time series and this result is particularly relevant for forecasting purpose. The more negative it is the value of ADF, stronger is the rejection of the hypothesis that the univariate time series has an unit root. Saying so, in this case the hypothesis of unit root is rejected for all the parameters, which make the time series estimation appropriate for forecasting.

As the Figure 3 confirms, the curvature factor represented by β_2 is the most volatile parameter, especially in years 2009 to the end of 2013. Since the existence of a "hump" in the yield curve can be interpreted as a predictor of financial instability and economic transition; the high volatility of parameter β_2 may be explained by the global financial crisis in July 2007 and by the impact of European Sovereign debt crisis, consequently by the austerity measures applied in Portugal at 2011.

The primary objective of this study is to extract the term structure of yield spread. Saying so, once the Portuguese yield curve is estimated, the yield spread curve can be fitted using the disjoint method. In Figure 4 is depicted the yield spread curve for varying maturities. Table A3 reports the descriptive statistics of yield spread for varying maturities.

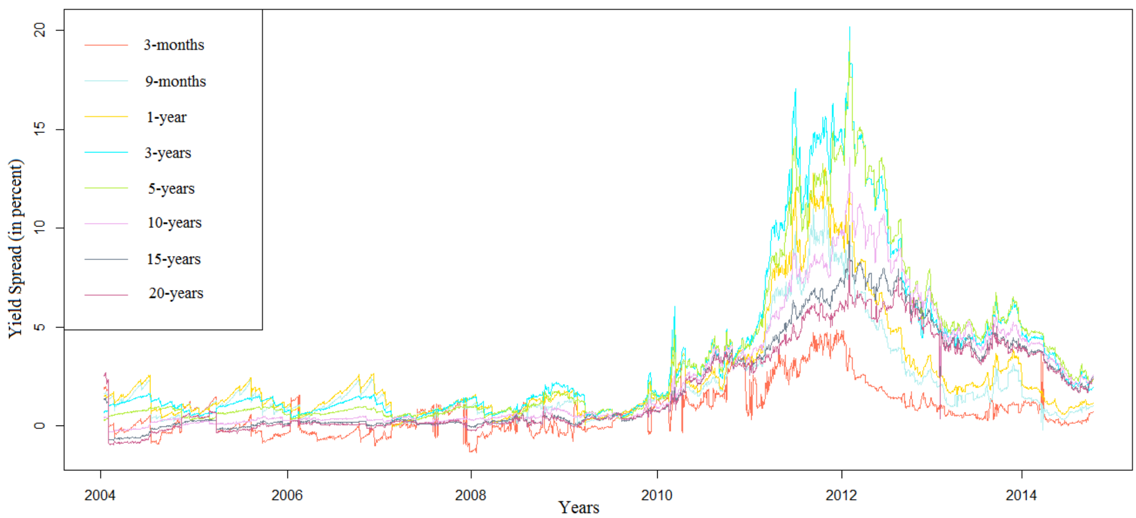


Figure 4: Portuguese Government Bonds Yield Spread over the period January 2004 through June 2014

In order to understand how the yield spread performs along the maturity spectrum, the maturities of short, medium and long run were chosen. By analyzing the

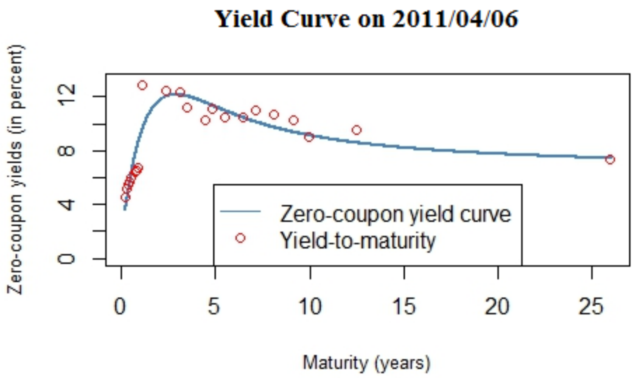
Table A3 in the appendix, we can see that, on average, the medium-term maturities yield spread is higher than the shorter term maturities; however, at about 3-years maturity the average yield spread tends to decrease as the maturity increases. Regarding the volatility of the mean yield spread, the medium-term maturities are more volatile than the shorter and longer term maturity.

Figure 4 depicts the evolution of yield spread over the period January 1^{st} 2004 to June 30^{th} 2014 for the maturities of 3-months, 9-months, 1-year, 3-years, 5-years, 10-years, 15-years and 20-years. Instead of depicts all the maturities analyzed in Table A3, we chose references maturities in order to ease the graphical analysis. We can see that until late of 2009, the yield spread has been following a stable pattern, with some seasonal fluctuations. Afterwards, the yield spread rose steadily and peaked in 2012. Taking into consideration the high level of yield spread experimented by Portugal during this period, Portugal Government revealed a statement in January 15^{th} 2010 to reassure to investors that the government is committed to reduce the deficit. However, during the period from 2010 through 2012, the Portuguese Government Bonds credit spread of maturities of three and five years have still changed significantly and reached a peak of about 20% in 2012. And during this period, several credit agency lowered Portugal's sovereign credit ranting by several notch. For instance, in July 13^{th} 2010 Moody's Investors Service lowered Portugal government bond rating from Aa2 to A1 and they pointed out the weak growth prospects faced by Portugal; in March 29^{th} 2011 Standards & Poor's downgrades the rating to BBB- from BBB; in April 1^{th} 2011 Fitch Ratings cutted the rating to BBB- from A-; in April 5^{th} 2011 Moody's Investors Service lowered the rating from A3 to Baa1.

The yield curve estimated from observed prices and the yield spread are strictly related. The Figure A1 in the appendix depicts the daily yield curve fit of year 2006, as we can see in this figure the yield remains between 1% to 4% (with some peaks reaching 5%); consequently, we can see in the Figure 4 that this period corresponds to relatively low yield spread. Likewise, we can see in the Figure A2 and A3 (which represent the daily yield curve fit of 2011 and 2012, respectively) that the yield is increasing over time and reaching a peak of about 20% at the beginning of 2012 and the Figure 4 shows that the yield spread follows the same pattern. Once again, through the analysis of the Figure A4 (daily fit of year 2014), we can see that the yield and credit spread are positively correlated.

Not only Portugal, but other countries such as Ireland, Greece, Italy and Spain have been affected by European Sovereign debt crisis, and consequently all of these countries experimented significant changes in their yield spread, especially during 2012.

The Figure 5 shows the impact of the financial turmoil faced and the impact of the austerity measures in Portugal. In April 6^{th} 2011 Portugal requested for a financial bailout from the European Union. As we can see in the first graph of the Figure 5, the yields are already relatively high at the time of the request due to the financial instability, Portugal's government announcement about budget cuts and a series of measures. In May 3^{rd} 2011 Portugal agreed with the European Union and the International Monetary Fund for a €72 billion financial bailout program. At the beginning of the program the yield spread increases significantly, especially yield spread of bonds with maturities of 3 and 5-years (as mentioned before, the yield



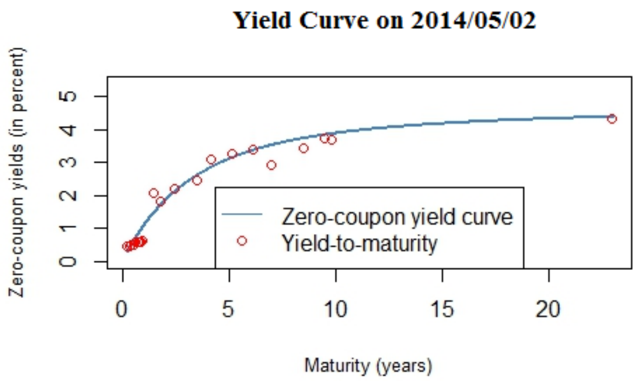


Figure 5: Impact of financial bailout on the yield curve

spread reached a peak about 20%). However, after several mission review during the last three years, the yield spread has been following a decreasing pattern. The second graph of the Figure 5 depicts the yield curve at the date of the final review mission to Portugal and, as we can see, the yield reduced significantly and tends to about 5% for long-term maturity Government bonds.

Moreover, it is also important to address the fitting errors. The price errors are calculated as the difference between market bonds prices and its theoretical price and the yield errors is computed as the difference between the bond yield and its theoretical yield. Figure A5 and A6 in the appendix depict the price errors and yields errors generated during the estimation procedure, respectively. Through the analysis of these two figures is noticed that the year that provided higher errors is 2012. These errors could be explained by different liquidity premium in the bond market due to the conditions faced by Portugal described above.

4.2 Out-of-sample Results

As mentioned in the Section 3.2.2, the yield curve is forecasted by predicting the estimated parameters of Nelson-Siegel model.

This research uses the Random Walk with drift as the benchmark forecast model and as the competitor models the AR(1) and VAR(1). For each model, we use the data from January 1^{st} 2004 to December 31^{th} 2013 and the forecast begins at January 1^{st} 2014 and go through June 30^{th} 2014 for the forecast horizon of 1-month, 3-months and 6-months.

Regarding the benchmark Random Walk with drift, the regressions estimated is presented in the Table 1 and the out-of-sample results can be found in Tables A4, A5 and A6 for time horizon of 1, 3 and 6-month ahead, respectively.

By analyzing the regressions of the Random Walk with drift, the positive non-zero constant term suggests that time series of $\beta_{0,t}$ and λ show an upward trend; otherwise, the $\beta_{1,t}$ and $\beta_{2,t}$ suggest a downward trend. All the estimated drift component is significant[1].

Relatively the AR(1), the regressions estimation are described in Table 2 and the out-of-sample results are shown in Tables A7, A8 and A9 for forecast horizon of 1, 3 and 6-month ahead, respectively.

[1]*** significant at 0%, ** significant at 0.001%and * significant at 0.01%

$\beta_{0,t} = \alpha_0 + \beta_{0,t-1}$

	Estimate	Std. Error	t-value	p-value
α_0	0.000923	0.010713	0.086148	$< 2.2\ e^{-16}$***

$\beta_{1,t} = \alpha_1 + \beta_{1,t-1}$

	Estimate	Std. Error	t-value	p-value
α_1	-0.000041	0.482021	-0.000085	0.000013***

$\beta_{2,t} = \alpha_2 + \beta_{2,t-1}$

	Estimate	Std. Error	t-value	p-value
α_2	-0.00725	0.494452	-0.014657	$< 2.2\ e^{-16}$***

$\lambda_t = \alpha_3 + \lambda_{t-1}$

	Estimate	Std. Error	t-value	p-value
α_3	0.000069	0.029321	0.000047	$< 2.2\ e^{-16}$***

Table 1: Regressions estimated for Random Walk with Drift

The stationarity of the these time-series is proved by the value estimated for ϕ_i, i=0,1,2,3, which is smaller than 1. Note that all the parameters estimated in AR(1) are significant.

Lastly, the Table 3 details the matrix of regressions for the VAR(1) model and the Tables A10 ,A11 and A12 display its out-of-sample results for the time horizon of 1, 3 and 6-months ahead.

To assess the out-of-sample results, we use the yield fitted by Nelson-Siegel model (1987) for the period of January 1^{st} 2004 through June 30^{th} 2014. Using these yields we computed the RMSE for the maturities of 3-months, 1-year, 3-years, 5-years and 10-years. The RMSE is computed as:

$$RMSE = \sqrt{\frac{1}{n}\sum_{t=1}^{n}(\hat{y}_t - y_t)^2} \tag{14}$$

which n stands for the number of observations, $\hat{y}_t$ stands for the forecasted yield and y_t stands for the yield extracted from the fitted yield curve.

By analyzing the RMSE, the out-of-sample results both AR(1) and VAR(1)

$\beta_{0,t} = \theta_0 + \phi_0 \beta_{0,t-1}$

	Estimate	Std. Error	t-value	p-value
θ_0	4.9474	0.1802	27.452	$< 2.2\ e^{-16***}$
ϕ_0	0.9418	0.066	143.6915	$< 2.2\ e^{-16***}$

$\beta_{1,t} = \theta_1 + \phi_1 \beta_{1,t-1}$

	Estimate	Std. Error	t-value	p-value
θ_1	-2.2424	0.968736	-2.3149	0.02063*
ϕ_1	0.5609	0.016205	34.6117	$2.2\ e^{-16***}$

$\beta_{2,t} = \theta_2 + \phi_2 \beta_{2,t-1}$

	Estimate	Std. Error	t-value	p-value
θ_2	6.6733	1.39658	-4.7783	$1.8\ e^{-6***}$
ϕ_2	0.6763	0.014413	46.9208	0.000***

$\lambda_t = \theta_3 + \phi_3 \lambda_{t-1}$

	Estimate	Std. Error	t-value	p-value
θ_3	2.1959	0.320877	6.8435	$< 7.7\ e^{-12***}$
ϕ_3	0.9111	0.008138	111.9497	$< 2.2\ e^{-16***}$

Table 2: Regressions estimated for AR(1)

are quite similar. Both, depending the maturity and the forecast horizon, may slightly outperform the results of the benchmark model. By comparing the RMSE of AR(1) and VAR(1), we can observe that AR(1) seems to produce lower errors as the maturity increases and this performance tends to improve as the forecasting time horizon increases. This results agree with the Diebold and Li (2006) conclusions, which argue that the forecasting models based on the Nelson-Siegel model perform well and this performance improves as the forecasting time-horizon increases.

Regarding the forecasting model selection criteria, Table A13 shows the values of log-likelihood, AIC and BIC (is a Schwarz criterion also known as Schwarz Information Criterion (SIC) or Schwarz-Bayesian Information Criterion) of each model.

$\beta_{0,t} = \alpha_0\beta_{0,t-1} + \theta_0\beta_{1,t-1} + \delta_0\beta_{2,t-1} + \gamma_0\lambda_{t-1}$

	Estimate	Std. Error	t-value	p-value
α_0	0.989374	0.002316	427.278	$< 2e^{-16}$ ***
θ_0	0.003648	0.000944	3.864	0.000114 ***
δ_0	0.003234	0.000787	4.111	$4.07e^{-5}$***
γ_0	0.009695	0.0029569	3.279	0.001057**

$\beta_{1,t} = \alpha_1\beta_{0,t-1} + \theta_1\beta_{1,t-1} + \delta_1\beta_{2,t-1} + \gamma_1\lambda_{t-1}$

	Estimate	Std. Error	t-value	p-value
α_1	-0.12760	0.09261	-1.378	0.168
θ_1	0.41969	0.03775	11.117	$< 2e^{-16}$***
δ_1	-0.13042	0.03146	-4.145	$3.5e^{-5}$***
γ_1	0.07444	0.11826	0.629	0.529

$\beta_{2,t} = \alpha_2\beta_{0,t-1} + \theta_2\beta_{1,t-1} + \delta_2\beta_{2,t-1} + \gamma_2\lambda_{t-1}$

	Estimate	Std. Error	t-value	p-value
α_2	0.18883	0.09522	1.983	0.0475*
θ_2	0.56526	0.03882	14.562	$<2e^{-16} * **$
δ_2	1.10561	0.03235	34.176	$<2e^{-16}$***
γ_2	-0.13615	0.12160	-1.120	0.2630

$\lambda_t = \alpha_3\beta_{0,t-1} + \theta_3\beta_{1,t-1} + \delta_3\beta_{2,t-1} + \gamma_3\lambda_{t-1}$

	Estimate	Std. Error	t-value	p-value
α_3	0.032492	0.006095	5.331	$1.06e^{-7}$***
θ_3	-0.00056	0.002485	-0.226	0.822
δ_3	-0.000532	0.002071	-0.257	0.797
γ_3	0.91817	0.007783	117.972	$< 2e^{-16}$***

Table 3: Regressions estimated for VAR(1)

The model that exhibits a higher log-likelihood and that minimizes AIC and BIC is considered the one which produces a better forecasts. Saying so, through the analyses of values displayed in Table A13, one concludes that AR(1) is the model that reveals a better forecast.

The underperformance of VAR(1) may be explained mainly by two reasons. First, the regressions are estimated by OLS and when the parameters are unconstrained they can assume any values that best fit the data. Second, since several economic time series are highly correlated with its own past values and with the past values of the other time series added in data set, a multicollinearity problem tends to increase as more time series and lagged values are added in the model. Besides that, the major drawback of VAR(1) is the fact that by increasing the number of parameters to be estimated, higher will be the number of variables of the model.

5 Conclusion

The primary aim of this research is to fit the term structure of yield spread, focusing on Portugal Government Bond over the period January 1^{st} 2004 through June 30^{th} 2014. To accomplish this objective, this study uses the disjoint method. Since the latter method requires as an input a defaultable term structure and a non-defaultable term structure, we estimated in daily basis the risky term-structure based on Portuguese Government Bonds daily prices and using the Nelson-Siegel model (1987). And we assumed as non-defaultable term structure the AAA-rated curve of euro area government central banks, estimated by ECB using the Svensson model (1994). Afterwards, the yield spread curve is obtained by the difference of these two curves.

By analyzing the fitting results, we can see that some stylized facts of yield curves are replicated in this research, such as: an upward sloping yield curve was the most frequent shape estimated; the yield curves assumed several of shapes throughout the time horizon (up sloping, humped and almost flat); some humped-shape curves show a *behavior of mean reversion*, because at some point, the volatility decreases as the maturity increases.

Regarding the Portuguese Government Bonds Yield Spread, we conclude that it followed a stable pattern, with some seasonal fluctuations, until the late of 2009. Hereafter, the yield spread experimented a significant change, especially between the end of 2009 to 2013. From the period 2009 to 2012, Portugal yield spread increased sharply and spread of bonds with maturity of 3 and 5-years reached a peak of about 20% at the beginning of 2012. Afterwards, it started to decrease gradually until 2014, where the yield spread values are close to pre-crisis values. Note that the period from 2009 to 2013 was precisely the years when Portugal faced a liquidity and debt crisis. This weak debt and financial position leaded Portugal asks for a €72 billion financial bailout from European Union and since then, Portugal was under an Economic and Financial Adjustment Programme. After several mission review, the final one was at May 2^{nd} and since then, the yield spread seems to stabilize.

Regarding the out-of-sample results, this research shows that the AR(1) and VAR(1) slightly outperform the benchmark Random Walk with drift model. This result agrees with the conclusion of Diebold and Li (2006), which argued that forecasts based on Nelson-Siegel model provides accurate results and this good performance increases as the forecasting time-horizon increases.We also computed the forecasting model selection criterions, which conclude that the AR(1) model provides a better fit than VAR(1).

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Appendix

Descriptive Statistics of Portuguese Government Bonds Yield Curve

Maturity	mean	stdev	skew	kurt	Min	Max
3-months	2.591	0.555	0.356	2.642	-0.159	6.436
6-months	3.190	3.786	0.498	3.473	-1.768	8.962
9-months	3.733	2.414	1.246	4.841	-0.130	11.523
1-year	4.164	2.413	1.599	3.240	0.726	19.154
15-months	4.499	2.629	1.778	3.214	1.030	14.911
18-months	4.762	2.834	1.952	3.628	1.280	16.279
21-months	4.970	3.003	2.052	3.819	1.393	17.592
2-years	5.135	3.138	2.108	3.893	1.504	18.648
30-months	5.375	3.319	2.154	3.882	1.716	20.131
3-years	5.534	3.409	2.157	3.798	1.917	20.972
4-years	5.707	3.416	2.120	3.600	2.286	21.441
5-years	5.776	3.300	2.062	3.385	2.614	20.928
6-years	5.793	3.136	1.991	3.132	2.906	20.042
7-years	5.786	2.959	1.911	2.838	3.001	19.025
8-years	5.766	2.787	1.824	2.513	3.050	18.015
9-years	5.741	2.627	1.732	2.175	3.112	17.073
10-years	5.716	2.483	1.640	1.938	3.184	16.224
15-years	5.610	1.978	1.216	0.473	3.261	13.258
20-years	5.549	1.718	0.901	-0.284	2.921	11.642

Table A1: Descriptive Statistics of Portuguese Government Bonds Yield Curve.The sample period is 2004:01 to 2014:06.

Descriptive Statistics of Estimated Parameters

Parameter	mean	stdev	min	max	ρ_1	ρ_{12}	ρ_{30}	ADF
$\beta_{0,t}$	5.522	3.041	7.048e-09	21.529	0.963	0.714	0.486	-5.273
$\beta_{1,t}$	-3.790	3.557	-1.771e+01	28.013	0.986	0.559	0.248	-6.694
$\beta_{2,t}$	5.216	14.702	-5.142e+01	62.634	0.973	0.674	0.194	-4.122
λ	2.842	3.937	6.134e-02	28.460	0.912	0.510	0.301	-6.2641

Table A2: Descriptive Statistics of Estimated Parameters, sample period: 2004:01 to 2014:06.

Descriptive Statistics of Portuguese Government Bonds Yield Spread

Maturity	mean	stdev	min	max	Skew	Kurt
3-months	0.759	1.231	-1.350	4.863	1.173	1.028
6-months	1.493	1.726	-1.840	8.415	1.922	3.162
9-months	2.076	2.226	-0.207	11.123	2.040	3.532
1-year	2.506	2.654	-0.013	13.055	2.039	3.455
15-months	2.823	3.008	0.006	14.391	2.007	3.261
18-months	3.056	3.295	0.033	15.944	1.966	3.047
21-months	3.228	3.524	0.065	17.194	1.924	2.845
2-years	3.353	3.704	0.095	18.182	1.884	2.665
30-months	3.504	3.944	0.159	19.519	1.812	2.373
3-years	3.566	4.066	0.217	20.204	1.751	2.153
4-years	3.535	4.093	0.305	20.318	1.653	1.833
5-years	3.400	3.970	0.285	19.510	1.569	1.574
6-years	3.224	3.786	0.202	18.319	1.491	1.327
7-years	3.038	3.586	0.108	17.025	1.416	1.079
8-years	2.858	3.391	-0.048	15.770	1.342	0.835
9-years	2.693	3.209	-0.189	14.617	1.271	0.598
10-years	2.544	3.046	-0.312	13.589	1.204	0.378
15-years	2.047	2.483	-0.724	10.135	0.932	-0.432
20-years	1.829	2.215	-0.949	8.451	0.749	-0.834

Table A3: Descriptive Statistics Portuguese Government Bonds of Yield Spread. Sample Period: 2004:01 to 2014:06.

Out-of-sample Results: Random Walk with Drift

Maturity	mean	stdev	RMSE	ρ_1	ρ_{12}
3-months	2.273	0.003	0.697	0.864	-0.251
1-year	0.911	0.004	1.927	0.728	-0.313
3-years	0.624	0.009	2.282	0.595	-0.395
5-years	1.687	0.007	2.070	0.466	-0.414
10-years	3.814	0.002	1.405	0.438	-0.360

Table A4: Out-of-sample Forecasting Results, RW with drift: 1-month ahead

Maturity	mean	stdev	RMSE	ρ_3	ρ_{18}
3-months	2.282	0.008	0.677	0.886	0.2611
1-year	0.899	0.011	1.930	0.863	0.222
3-years	0.595	0.025	2.289	0.729	0.146
5-years	1.662	0.022	2.076	0.641	0.184
10-years	3.805	0.007	1.400	0.596	0.146

Table A5: Out-of-sample Forecasting Results, RW with drift: 3-month ahead

Maturity	mean	stdev	RMSE	ρ_6	ρ_{24}
3-months	2.296	0.016	0.667	0.841	0.488
1-year	0.881	0.021	1.935	0.864	0.467
3-years	0.552	0.050	2.298	0.886	0.445
5-years	1.624	0.043	2.085	0.751	0.424
10-years	3.793	0.0145	1.412	0.774	0.403

Table A6: Out-of-sample Forecasting Results, RW with drift: 6-month ahead

Out-of-sample Results: AR(1)

Maturity	mean	stdev	RMSE	ρ_1	ρ_{12}
3-months	1.448	0.098	1.043	0.121	-0.151
1-year	3.721	0.124	1.630	0.720	-0.163
3-years	6.097	0.197	1.676	0.846	-0.248
5-years	6.715	0.209	1.601	0.324	-0.162
10-years	7.137	0.283	1.499	0.367	-0.171

Table A7: Out-of-sample Forecasting Results, AR(1): 1-month ahead

Maturity	mean	stdev	RMSE	ρ_3	ρ_{18}
3-months	1.547	0.087	1.089	0.851	0.176
1-year	3.547	0.151	1.576	0.811	0.154
3-years	5.743	0.303	1.567	0.831	0.172
5-years	6.350	0.347	1.482	0.838	0.181
10-years	6.6719	0.435	1.335	0.846	0.193

Table A8: Out-of-sample Forecasting Results, AR(1): 3-month ahead

Maturity	mean	stdev	RMSE	ρ_6	ρ_{24}
3-months	1.590	0.074	1.073	0.747	0.186
1-year	3.469	0.133	1.075	0.725	0.249
3-years	5.580	0.270	0.503	0.739	0.243
5-years	6.141	0.324	0.410	0.771	0.288
10-years	6.345	0.452	0.748	0.801	0.345

Table A9: Out-of-sample Forecasting Results, AR(1): 6-month ahead

Out-of-sample Results: VAR(1)

Maturity	mean	stdev	RMSE	ρ_1	ρ_{12}
3-months	1.243	0.026	0.940	0.709	-0.291
1-year	3.456	0.194	1.546	0.827	-0.264
3-years	6.051	0.193	1.662	0.835	-0.269
5-years	6.853	0.148	1.643	0.834	-0.268
10-years	7.321	0.168	1.559	0.851	-0.287

Table A10: Out-of-sample Forecasting Results, VAR(1): 1-month ahead

Maturity	mean	stdev	RMSE	ρ_3	ρ_{18}
3-months	1.339	0.087	0.989	0.911	0.211
1-year	3.368	0.132	1.518	0.636	-0.141
3-years	5.943	0.139	1.630	0.672	-0.125
5-years	6.761	0.107	1.615	0.669	-0.097
10-years	7.077	0.206	1.478	0.669	-0.097

Table A11: Out-of-sample Forecasting Results, VAR(1): 3-month ahead

Maturity	mean	stdev	RMSE	ρ_6	ρ_{24}
3-months	1.455	0.129	1.286	0.526	0.358
1-year	3.563	0.268	1.061	0.309	-0.039
3-years	5.968	0.457	0.135	0.086	0.002
5-years	6.622	0.514	0.649	-0.007	0.026
10-years	6.789	0.316	1.003	0.370	0.229

Table A12: Out-of-sample Forecasting Results, VAR(1): 6-month ahead

Forecasting Model Selection Criteria

Random Walk with drift				
Selection Criteria	$\beta_{1,t}$	$\beta_{2,t}$	$\beta_{3,t}$	λ
log-likelihood	-2152.85	-12034.08	-12100.38	-4595.53
AIC	4307.69	24070.15	24202.76	9193.06
BIC	4313.89	24076.02	24208.63	9198.9
AR(1)				
Selection Criteria	$\beta_{1,t}$	$\beta_{2,t}$	$\beta_{3,t}$	λ
log-likelihood	-2086.48	-11715.62	-11875.13	-4540.19
AIC	4178.96	23437.24	23756.26	9086.38
BIC	4196.55	23454.84	23773.86	9103.9
VAR(1)				
Selection Criteria	$\beta_{1,t}$	$\beta_{2,t}$	$\beta_{3,t}$	λ
log-likelihood	-24151.55			
AIC	48367.11			
BIC	48554.78			

Table A13: Forecasting Model Selection Criteria computed for the RW with drift, AR(1) and VAR(1).

Portuguese Government Bonds Daily Yield Curve Estimation

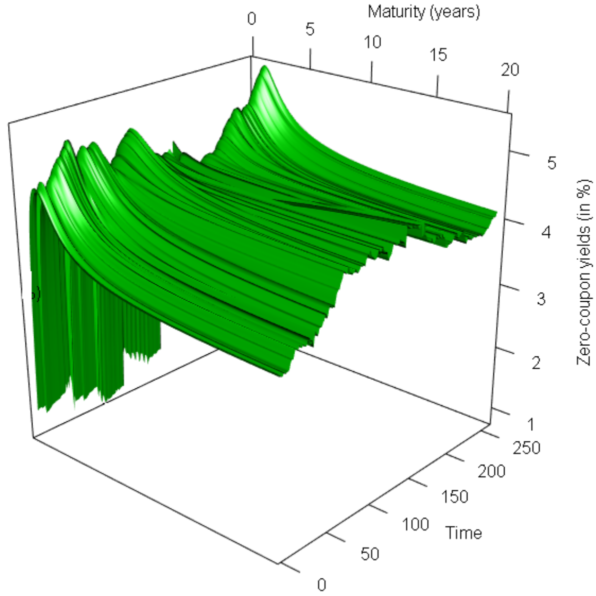

Figure A1: Portuguese Treasury Bonds Yield Curve daily fit of year 2006.

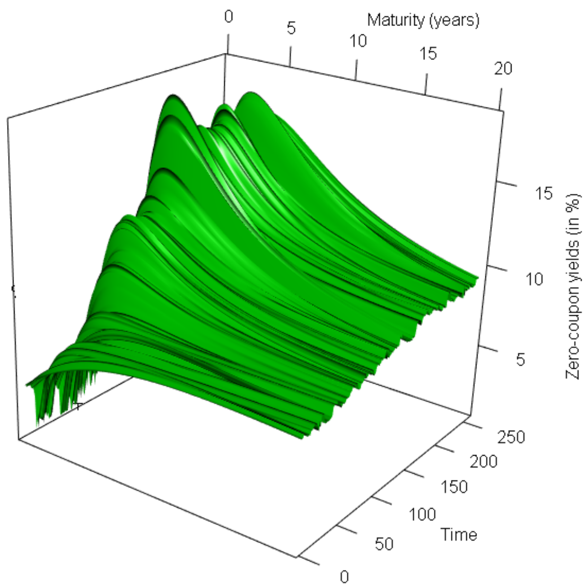

Figure A2: Portuguese Government Bonds Yield Curve daily fit of year 2011.

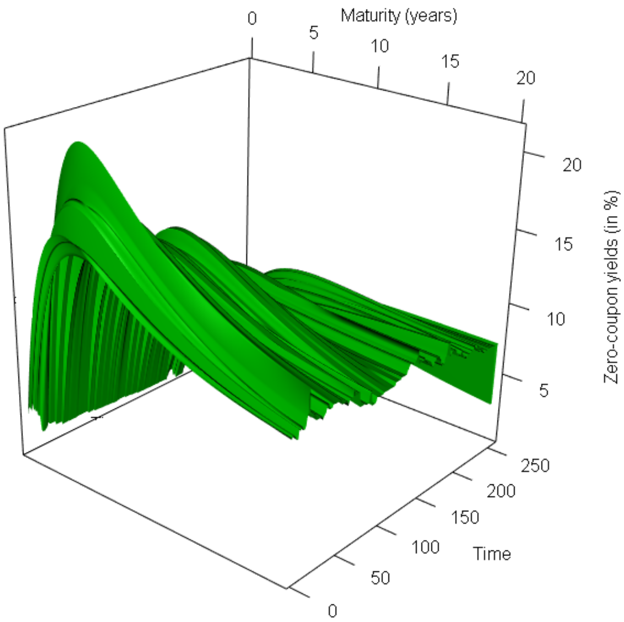

Figure A3: Portuguese Government Bonds Yield Curve daily fit of year 2012.

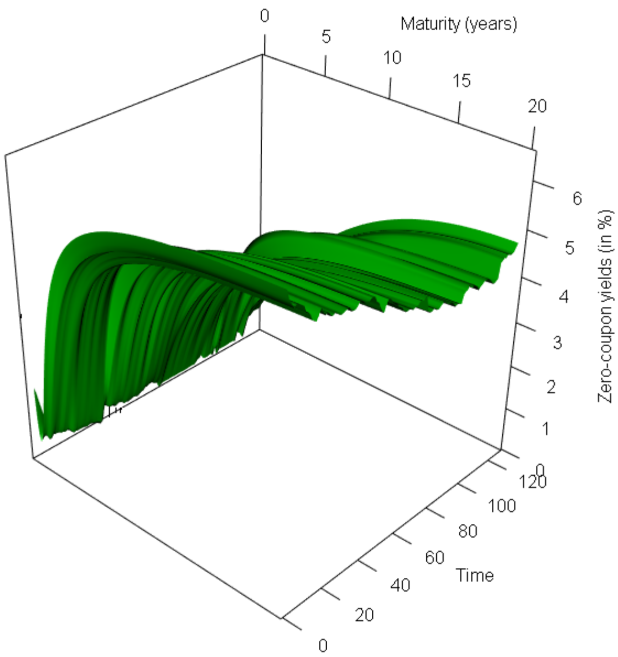

Figure A4: Portuguese Government Bonds Yield Curve daily fit of year 2014.

Price Errors of the Yield Curve Estimation

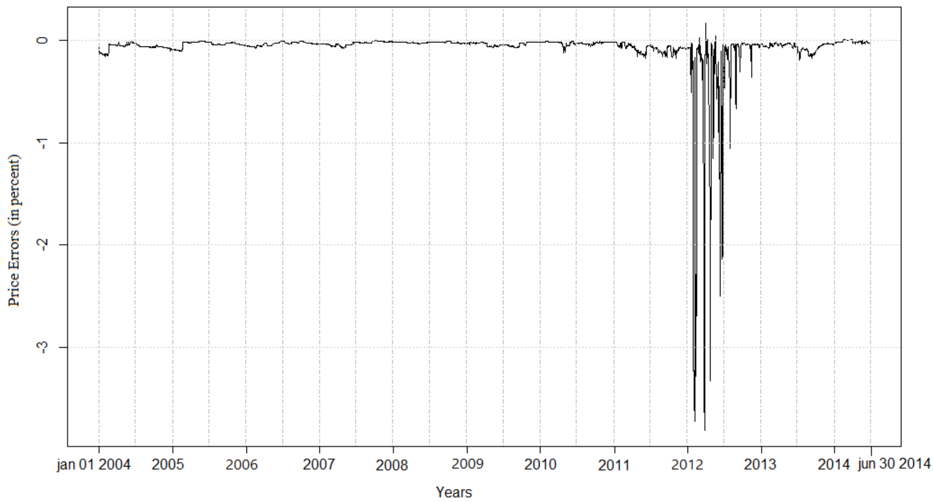

Figure A5: Pricing errors generated by estimating the yield curve over the period January 1^{st} 2004 through June 30^{th} 2014.

Yield Errors of the Yield Curve Estimation

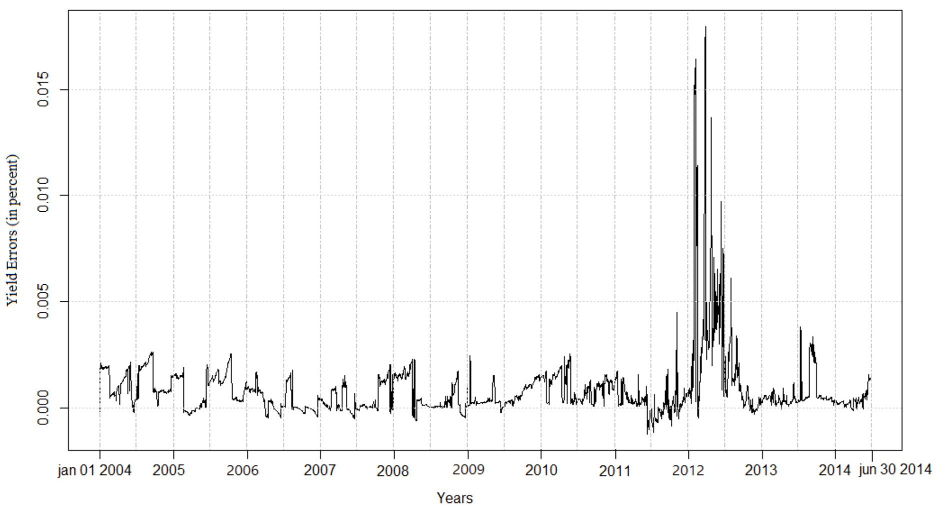

Figure A6: Yield errors generated by estimating the yield curve over the period January 1^{st} 2004 through June 30^{th} 2014.